The magnitude of the resultant shear force for the open section depends strongly on the peripherally weighted gradient $\partial\sigma_z(s,z)/\partial z$. An increase in the length of the boom leads to a decrease in this gradient because the $\sigma_z(s,z)$ distribution remains the same at the fixed end independent of the length. This implies that the magnitude of the thermal torque $T_{sc}(z)$ depends inversely on the length of the open section boom for each sun orientation.

By mechanical analogy, for a fixed/free open section boom: 1) for $CL^2/C_1 \ll 1$, the tip response $\phi(L) \simeq T_{sc}(z)\cdot L^3/3C_1$, hence $\phi(L) \simeq K_1L^2$, where K_1 is nearly a constant; 2) for $CL^2/C_1 \gg 1$, let $a = (CL^2/C_1)^{0.5}$. The mechanical torsion equation[8] for an open section boom is

$$T_{sc}(z) = C(d\phi/dz) - C_1(d^3\phi/dz^3)$$

The solution for a fixed/free boom can be obtained as

$$\phi(z) = \frac{T_{sc}(z)\cdot L}{C}\left\{\frac{Z}{L} - \frac{1}{a}\left[\sinh\frac{az}{L} - (\tanh a) \times \left(\cosh\frac{az}{L} - 1\right)\right]\right\}$$

for $CL^2/C_1 \gg 1$, implies $a \gg 1$ then $\phi(L) = (T_{sc}(z)\cdot L/C)$ $[1 - \text{Order }(1/a)]$ $\therefore$ the tip response $\phi(L) \rightarrow A$ constant, as $T_{sc}(z)$ is proportional to $1/L$.

Fixed/fixed boom

In an open section boom that is fixed at either end (i.e., prevented from warping), the stress distribution $\sigma_z(s,z)$ will be similar at both ends.

This implies that

$$\partial\sigma_z(s,z)/\partial z = 0 = \partial\tau(s,z)/\partial s$$

It would be expected that the shear stress, to satisfy the free edge boundary conditions will be uniformly zero at all points in the boom. Hence no twist should occur. This conclusion has also been reached by Jordan.[7]

Free/free boom

The thermal twist of an open section boom that is free to warp at either end may be regarded as two fixed/free booms, fixed symmetrically at the central plane. Relative to the central plane, the two halves of the boom twist in the same direction and manner as fixed/free open section booms, and the earlier discussion for a fixed/free boom becomes applicable. Boom sections equidistant from the central plane have zero angular displacement relative to each other.

Concluding Comments

The foregoing discussion used a circular open-section boom because of its conceptual simplicity for the illustration of thermally induced twist. The arguments and conclusions presented are equally applicable for other open-section booms. Actual flight booms (STEMs and BI-STEMs) have a certain amount of overlap whose physical properties of heat flow paths and interface friction may vary unpredictably along the length. However, because of their random nature, the assumption of their uniformity is justified as the boom length increases. Also, the interface friction introduces a resisting torque that the thermal torque $T_{sc}(z)$ must exceed before the boom behaves like an open-section one. The implication of this observation is that for fixed/free or free/free overlapped open-section booms greater than certain lengths, the thermal torque is negligibly small. Under the influence of a thermal torque that exceeds the friction torque, an open-section boom's response depends ideally on its end conditions. It is stressed that the suggested method of extrapolation from experimental results is accurate only for small angles of thermally induced twist, nominally, less than 45°.

Finally, on the basis of this article, a comment on the excellent paper by Frisch[9] is in order. The boundary condition $\partial^2\phi/\partial z^2|_{z=L} = 0$ (Eq. 54 of Ref. 9), is valid for the analysis, contrary to Jordan's[7] postulation for a fixed/free boom. The anomaly with the experimental evidence and Jordan's work lies in Eq. (1),[9] which has been applied with the implicit assumption that plane sections remain plane after the open-section thermally distorts. To satisfy the equilibrium conditions with which the expression for $\sigma_z(s,z)$ has been derived,[5] the boom will have already twisted. In this position, $\partial^2\phi/\partial z^2|_{z=L} = 0$ at the free end.

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Radiosity at the Midpoint of Parallel Plates

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Nomenclature

$B(X)$ = nondimensional radiosity
h = ratio of the plate separation distance to the plate length
H = parameter associated with the plate separation distance
$K(X,Y)$ = kernel of the Fredholm equation
L = length of the parallel plates
$R(X)$ = radiosity
T = temperature of the parallel plates
u = dummy variable of integration
γ = parallel plate separation distance
ϵ = emittance
ρ = reflectance
σ = Stefan-Boltzmann constant

Introduction

The Fredholm equation of the second kind that governs the radiative transfer between a parallel plate configuration does not lend itself to analytical treatment as the separation distance approaches zero. Approximate numerical approaches do not converge rapidly for this condition, especially when the emittance, ϵ, is small. This note is con-

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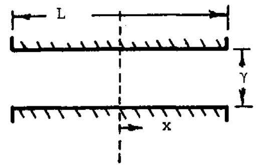
Fig. 1 Geometry of infinite length parallel plates.



cerned with the behavior of the solution at the midpoint as the separation distance approaches zero because the numerical techniques diverge under these conditions.

The parallel plate configuration is shown in Fig. 1. If both plates are identical in length, temperature, and surface properties (grey and diffuse) and are separated by a nonparticipating medium, the Fredholm equation can be cast in the nondimensional form[1]

$$B(X) = 1 + \lambda \int_{-1/2}^{1/2} B(Y)K(X,Y)dY \qquad (1)$$

where

$$X = x/L,\ B(X) = R(X)/\epsilon\sigma T^4,\ \lambda = \rho/2,\ h = \gamma/L$$

and

$$K(X,Y) = h^2/[(Y - X)^2 + h^2]^{3/2}$$

Using the approach of Rasmussen and Jischka,[2] an alternative form can be written by substituting $Y = uX$ and $h = HX$. This substitution allows h to be a variable for the analysis.

Table 1 Nondimensional radiosity at the midpoint (B_0) of infinite parallel plates

	$h = 0.05$		$h = 0.1$	
ϵ	B_0	$B(X = 0)$ exact	B_0	$B(X = 0)$ exact
0.1	9.056	9.077	6.137	7.205
0.3	3.280		3.052	3.095
0.5	1.988	1.988	1.944	1.942
0.7	1.425		1.414	1.411
0.9	1.1104	1.1104	1.108	1.107

$$B(X) = 1 + \lambda \int_{-1/2X}^{1/2X} B(uX)K(1,u)du \qquad (2)$$

The nondimensional radiosity at the midpoint, as γ approaches zero, is obtained by setting $X = 0$ in Eq. (2). Thus,

$$B(0) = 1 + \lambda B(0) \int_{-\infty}^{\infty} K(1,u)du \qquad (3)$$

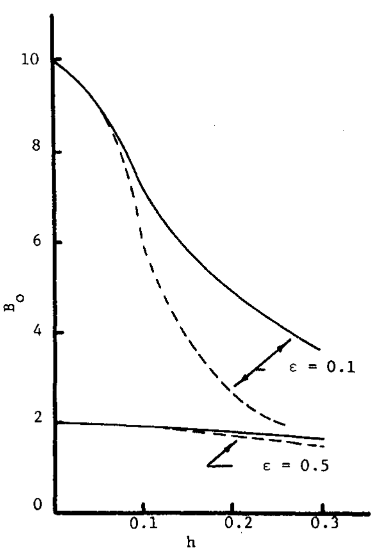


Fig. 2 Comparison of the magnitude of B_0 from Eq. (6) (---) and the exact value (——).

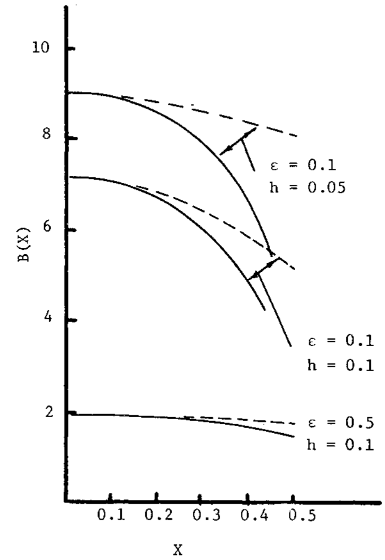


Fig. 3 Asymptotic behavior of Eq. (5) (---) and the exact values (——).

Straightforward integration yields

$$B(0) = 1/\epsilon \qquad (4)$$

as γ approaches zero. Thus, under the stated conditions the center point behaves like a blackbody.

Asymptotic Behavior

The asymptotic behavior of the radiosity, as γ and X approach zero, can be investigated by assuming that

$$B(X) = B_0 + B_1X^2,\ X \to 0 \qquad (5)$$

The constants B_0 and B_1 are determined by placing Eq. (5) in Eq. (2) and collecting and equating like powers of X asymptotically as X approaches 0. This procedure yields

$$B_0 = \frac{1/\epsilon}{1 + 2(\rho/\epsilon)h^2(1 - 3h^2) + (\rho h^2/\epsilon)(1 - 2h^2 + \frac{7}{3}\ln h)B_1/B_0} \qquad (6)$$

and

$$B_1 = -24(\rho/\epsilon)h^2/[1 + 2(\rho/\epsilon)h^2(5 - 12h^2)]B_0 \qquad (7)$$

Notice that Eq. (6) reduces to Eq. (4) as h approaches 0. This procedure is not to be confused with the variational method of Sparrow,[3] because it is a regular expansion technique in which the asymptotic behavior of ln terms must be dealt with carefully.

Figure 2 and Table 1 can be used to compare Eq. (6) with values given by Sparrow[3] and Love and Turner[4] when h and ϵ are the independent variables respectively. Figure 3 illustrates the form of Eq. (5). For the $h = 0.1$ case, the exact B_0 was used. Thus it appears that Eqs. (6) and (7) are appropriate asymptotic forms for the nondimensional radiosity as a function of ϵ, h, and X as the midpoint and separation distance of infinite parallel plates approach zero.

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